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REFLECTIVE POWER OF TWO-PHASE MEDIA OF CYLINDRICAL GEOMETRY

K. S. Adzerikho and N. V. Podluzhnyak

UDC 535.36

The brightness of the radiation reflected from a cylinder filled with particles of known optical properties is considered. The dependence of the reflective power on the optical properties of the medium and the experimental conditions is investigated in the single-scattering approximation. The limits of applicability of the method are estimated.

In determining the reflective power of two-phase media of cylindrical geometry, the approximation most commonly used is that of Eddington (see [1, 2], for example). As shown in [3, 4], it may correctly be used to calculate the emissive characteristics of two-phase media of nonplanar geometry. However, when external radiation is incident on a finite two-phase medium, the use of the Eddington approximation requires particular caution, especially for media of optical thickness $\tau \leqslant 1$-3. In the present work, the single-scattering approximation is used to calculate the reflective power of such media and its dependence on the optical properties of the medium and the experimental conditions is analyzed.

The solution of the radiation-transfer equation in a two-phase medium may be written in the form (see [5], for example)

$$I(s, l) = I(0, l)\exp\left[-\int_0^s \alpha(s')\,ds'\right] + \int_0^s J(s')\exp\left[-\int_{s'}^s \varkappa(s'')\,ds''\right] ds'. \tag{1}$$

Here I(s, l) is the radiation intensity at the point s in the direction $\mathbf{l} = \mathbf{l}(\theta, \varphi)$; I(0, l) is the intensity of the external radiation; J(s) is the emissive power of an elementary volume of the medium; $\alpha = \varkappa + \sigma$ is the attenuation coefficient, equal to the sum of the absorption and scattering coefficients.

Since J(s) depends on I(s, l) in the scattering medium, Eq. (1) may only be solved by numerical methods. Limiting consideration to the case of single (nonmultiple) scattering, a solution of the problem may be obtained by replacing J(s) in Eq. (1) by the distribution function for the sources created by the external radiation. In the case of nonplanar media, the integral term in Eq. (1) requires special consideration. Its physical meaning in the context of single scattering is fairly simple. It is the sum of the contributions of the radiation from each point of the medium in a given direction, taking into account attenuation.

Consider a medium of cylindrical geometry containing particles of known optical properties. The chosen coordinate system is shown in Fig. 1a: the x axis, from which η is measured is chosen in the plane containing the direction of the external radiation and the cylinder axis. The angles η and φ are positive when measured in the counterclockwise direction and negative in the opposite case. The angle θ, characterizing the direction of observation of the scattered radiation, is measured from the z axis. It is simple to show that, for normal incidence of the external radiation, the distribution of radiation sources in the cylindrical medium is given by the relation

$$S = S(r, \eta, l) = \frac{1}{\alpha} J = \frac{\lambda}{4\pi} p(\gamma) I_0 \exp[-T(r, \eta)], \tag{2}$$

where I_0 is the external-radiation intensity in the direction $\mathbf{l}_0 = \mathbf{l}_0(\theta_0, \varphi_0)$; $p(\gamma)$ is the scattering index for an elementary volume; γ is the angle between the incident and observed radiation; $\lambda = \sigma/(\varkappa + \sigma)$ is the probability of survival of a quantum; and:

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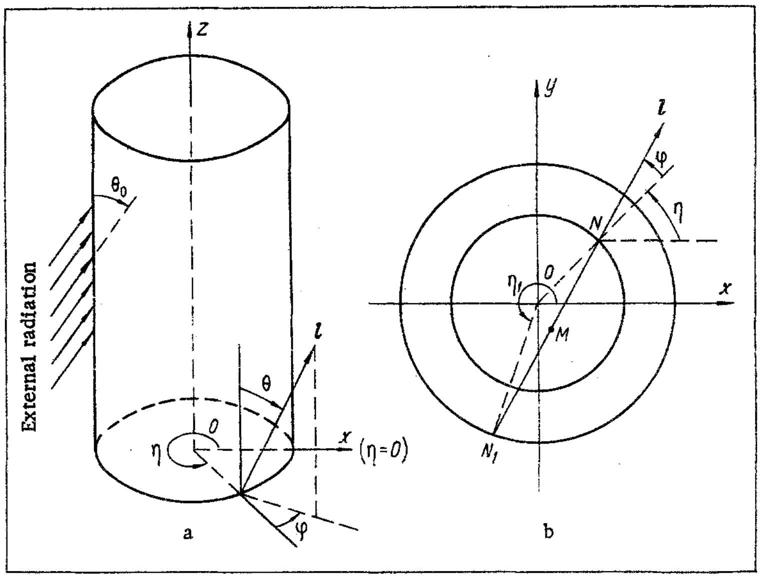

Fig. 1. Choice of coordinate system (a) and diagram of observation of scattered radiation (b).

$$T(r,\ \eta) = \alpha(\sqrt{R^2 - r^2 \sin^2 \eta} + r \cos \eta), \tag{3}$$

while R is the cylinder radius.

Note that incidence of the external radiation at some angle θ_0 to the cylinder surface may be taken into account if r is replaced by $r/\sin\theta_0$, and the reflective power depends not on θ and θ_0 but on γ.

In the case $\theta = 0$ or $\theta = \pi$ the solution may be obtained directly from the radiation-transfer equation written in a cylindrical coordinate system

$$I(\tau,\ \eta,\ \theta,\ \varphi)|_{\theta=0,\ \pi} = \frac{\lambda}{4\pi} p\left(\frac{\pi}{2}\right) I_0 \exp(-\sqrt{\tau_0^2 - \tau^2 \sin^2\eta} - \tau\cos\eta), \tag{4}$$

where $\alpha r = \tau$ and $\alpha R = \tau_0$ are the variable and total optical radii of the cylindrical medium.

Consider the case when the radiation scattered by a cylindrical medium is observed in a plane perpendicular to the cylinder axis (Fig. 1b). It follows from Fig. 1b that for an arbitrary direction of observation

$$\eta_1 = \eta + \arccos\left[\frac{\tau}{\tau_0}\sin^2\varphi - \cos\varphi\sqrt{1 - \left(\frac{\tau}{\tau_0}\sin\varphi\right)^2}\right], \tag{5}$$

$$\cos\eta_1 = -\cos(\eta + 2\varphi),\ \sin\eta_1 = -\sin(\eta + 2\varphi). \tag{6}$$

The intensity of the scattered radiation at some point $m = M(\tau'\cos\eta',\ \tau'\sin\eta')$ is

$$I_M = \rho_0 \exp\{-\sqrt{\tau_0^2 - y'^2} - x'\},$$

where $\rho_0 = (\lambda/4\pi)p(\gamma)I_0$ is a parameter characterizing the bulk scattering of the medium. Taking into account the contributions of the radiation from each point, attenuated over the path to the point N, the radiation at the point N is given in the single-scattering approximation by the expression

$$I(\tau,\ \eta,\ \theta,\ \varphi)|_{\theta=\frac{\pi}{2}} = \rho_0 \int_{\tau_0\cos\eta_1}^{\tau\cos\eta} \exp\{-\sqrt{\tau_0^2 - y'^2} - x' - |MN|\}\, dx'. \tag{7}$$

Writing the equation of the straight line N_1N in the form $y = b_1x + b_2$, where

$$b_1 = \frac{\tau\sin\eta - \tau_0\sin\eta_1}{\tau\cos\eta - \tau_0\cos\eta_1},\quad b_2 = \frac{\tau_0\tau\sin(\eta_1 - \eta)}{\tau\cos\eta - \tau_0\cos\eta_1}, \tag{8}$$

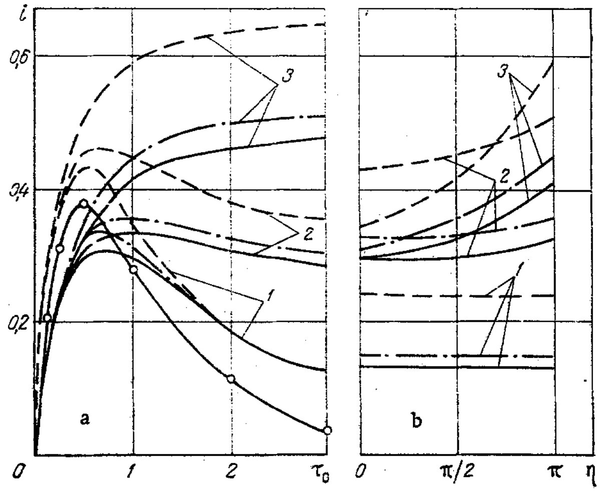

Fig. 2. Dependence of $i = \alpha I/2\rho_0\tau_0$ on the optical thickness τ_0((a) and its angular distribution (b): a) $\eta = 0$ (1), $\pi/2$ (2), π (3); b) $\tau_0 = 0.125$ (1), 0.5 (2), 1.0 (3); $\theta = \pi/2$ for the constant curve, $\pi/3$ for the dash-dot curve, and $\pi/6$ for the dashed curve.

Eq. (7) takes the form

$$I(\tau, \eta, \theta, \varphi)|_{\theta=\frac{\pi}{2}} = \rho_0 \int_{\tau_0 \cos\eta_1}^{\tau\cos\eta} \exp\{-\sqrt{\tau_0^2-(b_1x'+b_2)^2}-x'-b_3|\tau\cos\eta - x'|\}\,dx', \quad b_3=\sqrt{1+b_1^2}\,. \tag{9}$$

In considering the case $\theta \neq \pi/2$, it is necessary to take into account the change in the optical length of scattering; i.e., in Eq. (7) $|MN|$ must be replaced by $|MN|/\sin\theta$. Then the final expression for the intensity of the radiation propagating in a cylindrical medium, taking into account single-scattering processes, is*

$$I(\tau, \eta, \theta, \varphi) = I_0\delta(\mathbf{l}_0-\mathbf{l})\exp\{-\sqrt{\tau_0^2-\tau^2\sin^2\eta}-\tau\cos\eta\} + \\ + \frac{\lambda p(\gamma)}{4\pi\sin\theta} I_0 \int_{\tau_0\cos\eta_1}^{\tau\cos\eta} \exp\{-\sqrt{\tau_0^2-(b_1x'+b_2)^2}-x'-b_4|\tau\cos\eta-x'|\}\,dx', \tag{10}$$

where $b_4 = b_3/\sin\theta$.

The first term in this expression determines the attenuation by the cylindrical medium of the external radiation in the direction $\mathbf{l} = \mathbf{l}_0$.

When $\eta + \varphi \simeq \pi/2$, $3\pi/2$, i.e., for observation in directions close to the y axis, y must be used as the variable of integration. Then Eq. (10) is replaced by the following expression:

$$I(\tau, \eta, \theta, \varphi) = I_0\delta(\mathbf{l}_0-\mathbf{l})\exp\{-\sqrt{\tau_0^2-\tau^2\sin^2\eta}-\tau\cos\eta\} + \\ + \frac{\lambda p(\gamma)}{4\pi\sin\theta} I_0 \int_{\tau_0\sin\eta_1}^{\tau\sin\eta} \exp\{-\sqrt{\tau_0^2-y'^2}-(c_1y'+c_2)-c_3|\tau\sin\eta-y'|\}\,dy', \tag{10a}$$

where

$$c_1 = \frac{\tau\cos\eta-\tau_0\cos\eta_1}{\tau\sin\eta-\tau_0\sin\eta_1}; \quad c_2 = \frac{\tau_0\tau\sin(\eta-\eta_1)}{\tau\sin\eta-\tau_0\sin\eta_1}; \\ c_3 = \frac{\sqrt{1+c_1^2}}{\sin\theta}\,. \tag{11}$$

*This relation may be generalized without difficulty to the case of inhomogeneous media.

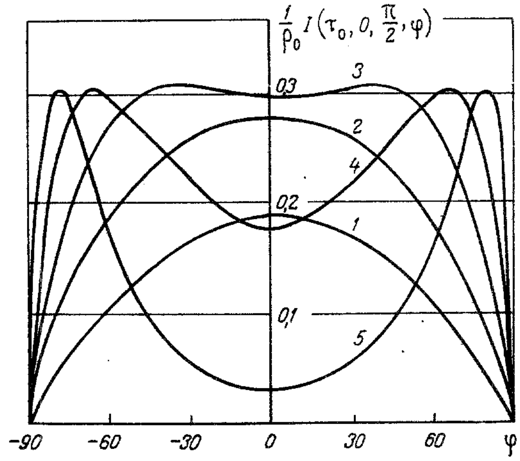

Fig. 3. Azimuthal distribution of intensity of singly scattered radiation for $\eta = 0$ ($\theta_0 = \theta = \pi/2$): 1) $\tau_0 = 0.125$; 2) 0.25; 3) 0.5; 4) 1.0; 5) 2.0.

Since scattered radiation is usually observed at distances much greater than the radius of the cylindrical medium, the determination of the total scattered-radiation intensity requires summation over φ in Eqs. (10) and (10a)

$$I(\tau_0, \eta, \theta) = \frac{1}{2N} \sum_{n=0}^{N} I\left(\tau_0, \eta \pm \frac{n\pi}{2N}, \theta, \mp \frac{n\pi}{2N}\right). \tag{12}$$

In the present calculations N = 9 was used. The dependence of the function $i = i(\tau_0, \eta, \theta) = (\alpha/2\rho_0\tau_0)I(\tau_0, \eta, \theta)$ on the optical radius τ_0 is shown in Fig. 2a. When the radiation is observed in the plane of incidence of the external radiation ($\eta = 0°$) at various angles θ, the intensity of the scattered radiation reaches a maximum close to $\tau_0 \sim 0.6$–0.8. For a plane layer $\tau_0 \sim 0.5$ (the curve marked by the unfilled circles) and in addition the change in intensity is sharper. In [6] it was shown that for $\lambda = 1$ the intensity of doubly scattered radiation is half that of singly scattered radiation for "reflection" and twice that of singly scattered radiation for "transmission." For transmission, its maximum is in the region $\tau_0 \sim 0.75$–0.80. Therefore, in calculating the reflective power of a two-phase medium restriction to the single-scattering approximation leads to an error of 25-30% for $\eta = 180°$. With decrease in $\lambda = \sigma/(\varkappa + \sigma)$, the error decreases in proportion to $\lambda/(2 + \lambda)$. In the case of transmission for $\lambda \sim 1$, it is necessary to restrict consideration to values $\tau_0 \sim 1$. For $\tau_0 > 1$ and $\eta = 0°$ the plane-layer approximation may be used to calculate the reflective power of a two-phase medium [5]. For observation from the side ($\eta = \pi/2$) there is a maximum, but for $\tau_0 \geq 2.5$–3.0 the intensity is almost unchanged.

The angular distribution of $i(\tau_0, \eta, \theta)$ when $\tau_0 < 1$ is almost spherical (Fig. 2b) and therefore the distribution of the light field in the single-scattering approximation will mainly be determined by the value of ρ_0, i.e., the scattering index for an elementary volume of the medium. For $\tau_0 \sim 1$, the degree of anisotropy of the scattered-radiation distribution

$$r = \frac{i(\eta = 0)}{i(\eta = \pi)}$$

reaches ~0.6, i.e., the fraction of back-scattered radiation becomes larger than in the direction of propagation of the external radiation.

In considering the dependence of $(1/\rho_0)I(\tau, \eta, \theta, \varphi)$ on φ, a fall in the scattered radiation with increase in φ is seen in the case of observation from the direction of the incident radiation. When $\eta = 0°$ the geometry of the scattering medium is significant: at the center ($\varphi = 0°$) there is a minimum even for $\tau_0 = 0.5$ because of the large attenuation (Fig. 3). The data shown in Fig. 3 also indicate that the results differ considerably from bulk scattering characterized by the parameter $\rho_0 = (\lambda/4\pi)p(\gamma)I_0$.

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NUMERICAL SOLUTION OF A NONSTEADY DIFFERENTIAL EQUATION OF HEAT CONDUCTION

V. M. Kapinos and Yu. L. Khrestovoi UDC 536.24.02

The use of a "floating" weight is suggested in the numerical solution of a parabolic differential equation of heat conduction with variable coefficients in integral-mean temperatures, used in the calculation of thermal expansions of turbine components. Recommendations are given for the determination of the optimum weights.

Heat-conduction problems which are reducible to one-dimensional problems, particularly in the calculation of the distribution of the integral-mean temperatures of turbine components for the determination of their thermal expansions [1, 2], lead to the following system of differential equations:

$$\frac{1}{a}\cdot\frac{\partial\vartheta}{\partial\tau} = L\vartheta + G(z,\ \tau),\ \vartheta = \vartheta(z,\ \tau),\ 0 \leqslant z \leqslant H,\ 0 \leqslant \tau \leqslant T; \quad (1)$$

$$\frac{\partial\vartheta}{\partial z} = \nu_0(\tau)\,\vartheta - \mu_0(\tau)\Big|_{z=0},\ \frac{\partial\vartheta}{\partial z} = -\nu_n(\tau)\vartheta + \mu_n(\tau)\Big|_{z=H}; \quad (2)$$

$$\vartheta|_{\tau=0} = \varphi_0(z), \quad (3)$$

where $L\vartheta = \partial^2\vartheta/\partial z^2 + A(z,\ \tau)\partial\vartheta/\partial z - B(z,\ \tau)\vartheta$, A, B, G, ν, and μ are assigned functions (B and $\nu > 0$); a is the coefficient of thermal diffusivity.

The system (1)-(3) will be solved numerically on the grid

$$\bar{\omega}_{h\Delta\tau} = \bar{\omega}_h \times \bar{\omega}_{\Delta\tau} = \{(ih,\ j\Delta\tau),\ i = 0,\ 1,\ 2,\ \ldots,\ n,\quad j = 0,\ 1,\ 2,\ \ldots,\ j_m\} \quad (4)$$

with steps $h = H/n$ and $\Delta\tau = T/j_m$.

Designating the value of the unknown grid function at the node $(z_{i,j})$ as $\theta_{i,j}$ and introducing the required number η of real parameters, also grid functions in the general case, we obtain a parametric family of difference schemes approximating the system (1)-(3).

The approximation of Eq. (1) on a six-point pattern is written as

$$\frac{1}{a}\,\theta_\tau = \Lambda^*\theta_\eta + \tilde{G}|_i,\ 0 < i < n,\quad 0 \leqslant j \leqslant j_m, \quad (5)$$

where $\theta_\tau = (\theta_{j+1} - \theta_j)\Delta\tau$, $\Lambda^* = \Lambda + \tilde{A}l - \tilde{B}$, $\Lambda\theta_i = (\theta_{i+1} - 2\theta_i + \theta_{i-1})/h^2$, $l\theta_i = (\theta_{i+1} - \theta_{i-1})/2h$ are linear operators while $\theta_\eta = \eta\theta_{j+1} + (1 - \eta)\theta_j$.

The coefficients of Eq. (1) are determined for each time layer with its weight

$$\tilde{X} = \eta_X X_{j+1} + (1 - \eta_X)\,X_j,\ X = A,\ B,\ G. \quad (6)$$

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